Functions
In Exercises 1–6, find the domain and range of each function.

1. \( f(x) = 1 + x^2 \)
2. \( f(x) = 1 - \sqrt{x} \)
3. \( F(t) = \frac{1}{\sqrt{t}} \)
4. \( F(t) = \frac{1}{1 + \sqrt{t}} \)
5. \( g(z) = \sqrt{4 - z^2} \)
6. \( g(z) = \frac{1}{\sqrt{4 - z^2}} \)

In Exercises 7 and 8, which of the graphs are graphs of functions of \( x \), and which are not? Give reasons for your answers.

7. a. b.

8. a. b.

9. Consider the function \( y = \sqrt{1/x} - 1 \).
   a. Can \( x \) be negative?
   b. Can \( x = 0 \)?
   c. Can \( x \) be greater than 1?
   d. What is the domain of the function?

10. Consider the function \( y = \sqrt{2 - \sqrt{x}} \).
    a. Can \( x \) be negative?
    b. Can \( \sqrt{x} \) be greater than 2?
    c. What is the domain of the function?

Finding Formulas for Functions
11. Express the area and perimeter of an equilateral triangle as a function of the triangle’s side length \( x \).

12. Express the side length of a square as a function of the length \( d \) of the square’s diagonal. Then express the area as a function of the diagonal length.

13. Express the edge length of a cube as a function of the cube’s diagonal length \( d \). Then express the surface area and volume of the cube as a function of the diagonal length.

14. A point \( P \) in the first quadrant lies on the graph of the function \( f(x) = \sqrt{x} \). Express the coordinates of \( P \) as functions of the slope of the line joining \( P \) to the origin.

Functions and Graphs
Find the domain and graph the functions in Exercises 15–20.

15. \( f(x) = 5 - 2x \)
16. \( f(x) = 1 - 2x - x^2 \)
17. \( g(x) = \sqrt{|x|} \)
18. \( g(x) = \sqrt{-x} \)
19. \( F(t) = t/|t| \)
20. \( G(t) = 1/|t| \)

21. Graph the following equations and explain why they are not graphs of functions of \( x \).
   a. \( |y| = x \)
   b. \( y^2 = x^2 \)

22. Graph the following equations and explain why they are not graphs of functions of \( x \).
   a. \( |x| + |y| = 1 \)
   b. \( |x + y| = 1 \)

Piecewise-Defined Functions
Graph the functions in Exercises 23–26.

23. \( f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 2 - x & 1 < x \leq 2 \end{cases} \)
24. \( g(x) = \begin{cases} 1 - x & 0 \leq x \leq 1 \\ 2 - x & 1 < x \leq 2 \end{cases} \)

25. \( F(x) = \begin{cases} 3 - x & x \leq 1 \\ 2x & x > 1 \end{cases} \)
26. \( G(x) = \begin{cases} 1/x & x < 0 \\ x & 0 \leq x \end{cases} \)

27. Find a formula for each function graphed.
28. a. 

![Graph](image)

b. 

![Graph](image)

29. a. 

![Graph](image)

b. 

![Graph](image)

30. a. 

![Graph](image)

b. 

![Graph](image)

31. a. Graph the functions \( f(x) = x/2 \) and \( g(x) = 1 + (4/x) \) together to identify the values of \( x \) for which 

\[
\frac{x}{2} > 1 + \frac{4}{x}.
\]

b. Confirm your findings in part (a) algebraically.

32. a. Graph the functions \( f(x) = 3/(x - 1) \) and \( g(x) = 2/(x + 1) \) together to identify the values of \( x \) for which 

\[
\frac{3}{x - 1} < \frac{2}{x + 1}.
\]

b. Confirm your findings in part (a) algebraically.

**The Greatest and Least Integer Functions**

33. For what values of \( x \) is

a. \( \lfloor x \rfloor = 0 ? \)

b. \( \lceil x \rceil = 0 ? \)

34. What real numbers \( x \) satisfy the equation \( \lfloor x \rfloor = \lceil x \rceil \)?

35. Does \( \lfloor -x \rfloor = -\lceil x \rceil \) for all real \( x \)? Give reasons for your answer.

36. Graph the function

\[
f(x) = \begin{cases} 
\lfloor x \rfloor, & x \geq 0 \\
\lceil x \rceil, & x < 0 
\end{cases}
\]

Why is \( f(x) \) called the integer part of \( x \)?

**Theory and Examples**

37. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 14 in. by 22 in. by cutting out equal squares of side \( x \) at each corner and then folding up the sides as in the figure. Express the volume \( V \) of the box as a function of \( x \).

![Box Diagram](image)

38. The figure shown here shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.

a. Express the \( y \)-coordinate of \( P \) in terms of \( x \). (You might start by writing an equation for the line \( AB \).)

b. Express the area of the rectangle in terms of \( x \).

39. A cone problem  Begin with a circular piece of paper with a 4 in. radius as shown in part (a). Cut out a sector with an arc length of \( x \). Join the two edges of the remaining portion to form a cone with radius \( r \) and height \( h \), as shown in part (b).

![Cone Diagram](image)

a. Explain why the circumference of the base of the cone is \( 8\pi - x \).

b. Express the radius \( r \) as a function of \( x \).

c. Express the height \( h \) as a function of \( x \).

d. Express the volume \( V \) of the cone as a function of \( x \).
40. **Industrial costs** Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs $180 per foot across the river and $100 per foot along the land.

a. Suppose that the cable goes from the plant to a point $Q$ on the opposite side that is $x$ ft from the point $P$ directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance $x$.

b. Generate a table of values to determine if the least expensive location for point $Q$ is less than 2000 ft or greater than 2000 ft from point $P$.

41. For a curve to be **symmetric about the x-axis**, the point $(x, y)$ must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x-axis is not the graph of a function, unless the function is $y = 0$.

42. **A magic trick** You may have heard of a magic trick that goes like this: Take any number. Add 5. Double the result. Subtract 6. Divide by 2. Subtract 2. Now tell me your answer, and I’ll tell you what you started with. Pick a number and try it.

You can see what is going on if you let $x$ be your original number and follow the steps to make a formula $f(x)$ for the number you end up with.