Chapter 1 Practice Exercises

Inequalities
In Exercises 1–4, solve the inequalities and show the solution sets on the real line.

1. \( 7 + 2x \geq 3 \)
2. \( -3x < 10 \)
3. \( \frac{1}{5}(x - 1) < \frac{1}{4}(x - 2) \)
4. \( \frac{x - 3}{2} \geq -\frac{4 + x}{3} \)

Absolute Value
Solve the equations or inequalities in Exercises 5–8.

5. \( |x + 1| = 7 \)
6. \( |y - 3| < 4 \)
7. \( \left| 1 - \frac{x}{2} \right| > \frac{3}{2} \)
8. \( \left| \frac{2x + 7}{3} \right| \leq 5 \)

Coordinates
9. A particle in the plane moved from \( A(-2, 5) \) to the y-axis in such a way that \( \Delta y \) equaled \( 3 \Delta x \). What were the particle’s new coordinates?
10. a. Plot the points \( A(8, 1), B(2, 10), C(-4, 6), D(2, -3), \) and \( E(14/3, 6) \).
   b. Find the slopes of the lines \( AB, BC, CD, DA, CE, \) and \( BD \).
   c. Do any four of the five points \( A, B, C, D, \) and \( E \) form a parallelogram?
   d. Are any three of the five points collinear? How do you know?
   e. Which of the lines determined by the five points pass through the origin?
11. Do the points \( A(6, 4), B(4, -3), \) and \( C(-2, 3) \) form an isosceles triangle? A right triangle? How do you know?
12. Find the coordinates of the point on the line \( y = 3x + 1 \) that is equidistant from \( (0, 0) \) and \( (-3, 4) \).

Lines
In Exercises 13–24, write an equation for the specified line.

13. through \((1, -6)\) with slope 3
14. through \((-1, 2)\) with slope \(-1/2\)
15. the vertical line through \((0, -3)\)
16. through \((-3, 6)\) and \((1, -2)\)
17. the horizontal line through \((0, 2)\)
18. through \((3, 3)\) and \((-2, 5)\)
19. with slope \(-3\) and y-intercept 3
20. through \((3, 1)\) and parallel to \(2x - y = -2\)
21. through \((4, -12)\) and parallel to \(4x + 3y = 12\)
22. through \((-2, -3)\) and perpendicular to \(3x - 5y = 1\)
23. through \((-1, 2)\) and perpendicular to \((1/2)x + (1/3)y = 1\)
24. with x-intercept 3 and y-intercept -5

Functions and Graphs
25. Express the area and circumference of a circle as functions of the circle’s radius. Then express the area as a function of the circumference.
26. Express the radius of a sphere as a function of the sphere’s surface area. Then express the surface area as a function of the volume.
27. A point \( P \) in the first quadrant lies on the parabola \( y = x^2 \). Express the coordinates of \( P \) as functions of the angle of inclination of the line joining \( P \) to the origin.
28. A hot-air balloon rising straight up from a level field is tracked by a range finder located 500 ft from the point of liftoff. Express the balloon’s height as a function of the angle the line from the range finder to the balloon makes with the ground.

In Exercises 29–32, determine whether the graph of the function is symmetric about the y-axis, the origin, or neither.

29. \( y = x^{1/5} \)
30. \( y = x^{2/3} \)
31. \( y = x^2 - 2x - 1 \)
32. \( y = e^{-x^2} \)

In Exercises 33–40, determine whether the function is even, odd, or neither.

33. \( y = x^2 + 1 \)
34. \( y = x^5 - x^3 - x \)
35. \( y = 1 - \cos x \)
36. \( y = \sec x \tan x \)
37. \( y = \frac{x^4 + 1}{x^3 - 2x} \)
38. \( y = 1 - \sin x \)
39. \( y = x + \cos x \)
40. \( y = \sqrt{x^4 - 1} \)
In Exercises 41–50, find the (a) domain and (b) range.

41. \( y = |x| - 2 \)  
42. \( y = -2 + \sqrt{1 - x} \)  
43. \( y = \sqrt{16 - x^2} \)  
44. \( y = 3^{2-x} + 1 \)  
45. \( y = 2e^{-x} - 3 \)  
46. \( y = \tan(2x - \pi) \)  
47. \( y = 2\sin(3x + \pi) - 1 \)  
48. \( y = x^{2/3} \)  
49. \( y = \ln(x - 3) + 1 \)  
50. \( y = -1 + \sqrt{2 - x} \)

**Piecewise-Defined Functions**

In Exercises 51 and 52, find the (a) domain and (b) range.

51. \( y = \begin{cases} \sqrt{-x}, & -4 \leq x \leq 0 \\ \sqrt{x}, & 0 < x \leq 4 \end{cases} \)  
52. \( y = \begin{cases} -x - 2, & -2 \leq x \leq -1 \\ x, & -1 < x \leq 1 \\ x + 2, & 1 < x \leq 2 \end{cases} \)

In Exercises 53 and 54, write a piecewise formula for the function.

53.  
54.  

**Composition of Functions**

In Exercises 55 and 56, find

a. \((f \circ g)(-1)\)  
b. \((g \circ f)(2)\)  
c. \((f \circ f)(x)\)  
d. \((g \circ g)(x)\)

55. \( f(x) = \frac{1}{x}, \quad g(x) = \frac{1}{\sqrt{x} + 2} \)  
56. \( f(x) = 2 - x, \quad g(x) = \sqrt{x} + 1 \)

In Exercises 57 and 58, (a) write a formula for \(f \circ g\) and \(g \circ f\) and find the (b) domain and (c) range of each.

57. \( f(x) = 2 - x^2, \quad g(x) = \sqrt{x} + 2 \)  
58. \( f(x) = \sqrt{x}, \quad g(x) = \sqrt{1 - x} \)

**Composition with absolute values**

In Exercises 59–64, graph \( f_1 \) and \( f_2 \) together. Then describe how applying the absolute value function before applying \( f_1 \) affects the graph.

59. \( f_1(x) = \frac{1}{x}, \quad f_2(x) = f_1(|x|) \)  
60. \( x^3, \quad |x|^3 \)  
61. \( x^2, \quad |x|^2 \)  
62. \( \frac{1}{x}, \quad \frac{1}{|x|} \)  
63. \( \sqrt{x}, \quad \sqrt{|x|} \)  
64. \( \sin x, \quad \sin |x| \)

**Trigonometry**

In Exercises 65–68, graph \( g_1 \) and \( g_2 \) together. Then describe how taking absolute values after applying \( g_1 \) affects the graph.

65. \( g_1(x) = \frac{x}{|x|} \)  
66. \( \sqrt{x}, \quad |\sqrt{x}| \)  
67. \( 4 - x^2, \quad |4 - x^2| \)  
68. \( x^2 + x, \quad |x^2 + x| \)

**Height of a pole**

Two wires stretch from the top \( T \) of a vertical pole to points \( B \) and \( C \) on the ground, where \( C \) is 10 m closer to the base of the pole than is \( B \). If wire \( BT \) makes an angle of 35° with the horizontal and wire \( CT \) makes an angle of 50° with the horizontal, how high is the pole?

**Height of a weather balloon**

Observers at positions \( A \) and \( B \) 2 km apart simultaneously measure the angle of elevation of a weather balloon to be 40° and 70°, respectively. If the balloon is directly above a point on the line segment between \( A \) and \( B \), find the height of the balloon.

81. a. Graph the function \( f(x) = \sin x + \cos(x/2) \).
   b. What appears to be the period of this function?
   c. Confirm your finding in part (b) algebraically.
82. a. Graph \( f(x) = \sin(1/x) \).
   b. What are the domain and range of \( f' \)?
   c. Is \( f \) periodic? Give reasons for your answer.