Chapter 9

Practice Exercises

In Exercises 1–20 solve the differential equation.

1. \( \frac{dy}{dx} = \sqrt{y} \cos^2 \sqrt{y} \)
2. \( y' = \frac{3y(x + 1)^2}{y - 1} \)
3. \( yy' = \sec y^2 \sec^2 x \)
4. \( y \cos^2 x \frac{dy}{dx} + \sin x \frac{dy}{dx} = 0 \)
5. \( y' = xe^{1/2} \)
6. \( y' = x \sin x \)
7. \( \sec x y' + x \cos^2 y \frac{dy}{dx} = 0 \)
8. \( 2x^2 \frac{dy}{dx} - 3 \sqrt{y} \csc x \frac{dy}{dx} = 0 \)
9. \( y' = \frac{e^y}{x^2} \)
10. \( y' = xe^{-y} \csc y \)
11. \( x(x - 1) \frac{dy}{dx} - y \frac{dy}{dx} = 0 \)
12. \( y' = (y^2 - 1)x^{-1} \)
13. \( 2y' - y = xe^{1/2} \)
14. \( y' + y = e^{-x} \sin x \)
15. \( xy' + 2y = 1 - x^{-1} \)
16. \( xy' - y = 2x \ln x \)
17. \( (1 + e^x) \frac{dy}{dx} + (ye^x + e^{-x}) \frac{dy}{dx} = 0 \)
18. \( e^{-x} \frac{dy}{dx} + (e^{-x} y - 4x) \frac{dy}{dx} = 0 \)
19. \( (x + 3y^2) \frac{dy}{dx} + y \frac{dy}{dx} = 0 \) \( (Hint: \frac{d}{dx}(xy) = y \frac{dy}{dx} + x \frac{dy}{dx}) \)
20. \( x \frac{dy}{dx} + (3y - x^{-2} \cos x) \frac{dy}{dx} = 0, \quad x > 0 \)

Initial Value Problems

In Exercises 21–30 solve the initial value problem.

21. \( \frac{dy}{dx} = e^{-x-y} - 2, \quad y(0) = -2 \)
22. \( \frac{dy}{dx} = \frac{y \ln y}{1 + x^2}, \quad y(0) = e^2 \)
23. \( (x + 1) \frac{dy}{dx} + 2y = x, \quad x > -1, \quad y(0) = 1 \)
24. \( x \frac{dy}{dx} + 2y = x^2 + 1, \quad x > 0, \quad y(1) = 1 \)
25. \( \frac{dy}{dx} + 3x^2 y = x^2, \quad y(0) = -1 \)
26. \( x \frac{dy}{dx} + (y - \cos x) \frac{dy}{dx} = 0, \quad y\left(\frac{\pi}{2}\right) = 0 \)
27. \( x \frac{dy}{dx} + (y + \sqrt{y}) \frac{dy}{dx} = 0, \quad y(1) = 1 \)
28. \( y^{-2} \frac{dx}{dy} = \frac{e^y}{e^{2x} + 1}, \quad y(0) = 1 \)
29. \( xy' + (x - 2)y = 3x^2 e^{-x}, \quad y(1) = 0 \)
30. \( y \frac{dx}{dy} + (3x - xy + 2) \frac{dy}{dx} = 0, \quad y(2) = -1, \quad y < 0 \)

Euler's Method

In Exercises 31 and 32, use the stated method to solve the initial value problem on the given interval starting at \( x_0 \) with \( \Delta x = 0.1 \).

T 31. Euler: \( y' = y + \cos x, \quad y(0) = 0; \quad 0 \leq x \leq 2; \quad x_0 = 0 \)

T 32. Improved Euler: \( y' = (2 - y)(2x + 3), \quad y(-3) = 1; \quad -3 \leq x \leq -1; \quad x_0 = -3 \)

In Exercises 33 and 34, use the stated method with \( \Delta x = 0.05 \) to estimate \( y(c) \) where \( y \) is the solution to the given initial value problem.

T 33. Improved Euler:\n\[ c = 3; \quad \frac{dy}{dx} = \frac{x - 2y}{x + 1}, \quad y(0) = 1 \]
34. Euler:
\[ c = 4; \quad \frac{dy}{dx} = \frac{x^2 - 2y + 1}{x}, \quad y(1) = 1 \]

In Exercises 35 and 36, use the stated method to solve the initial value problem graphically, starting at with

- a. \( dx = 0.1 \)
- b. \( dx = -0.1 \)

35. Euler:
\[ \frac{dy}{dx} = \frac{1}{e^{x+y} + 2}, \quad y(0) = -2 \]

36. Improved Euler:
\[ \frac{dy}{dx} = -\frac{x^2 + y}{e^x + x^2}, \quad y(0) = 0 \]

Slope Fields
In Exercises 37–40, sketch part of the equation's slope field. Then add to your sketch the solution curve that passes through the point \( P(1, -1) \). Use Euler's method with \( x_0 = 1 \) and \( dx = 0.2 \) to estimate \( y(2) \). Round your answers to four decimal places. Find the exact value of \( y(2) \) for comparison.

- 37. \( y' = x \)
- 38. \( y' = 1/x \)
- 39. \( y'' = xy \)
- 40. \( y' = 1/y \)

Autonomous Differential Equations and Phase Lines
In Exercises 41 and 42.

- a. Identify the equilibrium values. Which are stable and which are unstable?
- b. Construct a phase line. Identify the signs of \( y' \) and \( y'' \).
- c. Sketch a representative selection of solution curves.

41. \( \frac{dy}{dx} = y^2 - 1 \)
42. \( \frac{dy}{dx} = y - y^2 \)

Applications

43. Escape velocity The gravitational attraction \( F \) exerted by an airless moon on a body of mass \( m \) at a distance \( s \) from the moon’s center is given by the equation \( F = -\frac{mg}{R^2}s^{-2} \), where \( g \) is the acceleration of gravity at the moon’s surface and \( R \) is the moon’s radius (see accompanying figure). The force \( F \) is negative because it acts in the direction of decreasing \( s \).

\[ F = -\frac{mgR^2}{s^2} \]

44. Coasting to a stop Table 9.9 shows the distance \( s \) (meters) coasted on in-line skates in \( t \) sec by Johnathon Krueger. Find a model for his position in the form of Equation (2) of Section 9.5. His initial velocity was \( v_0 = 0.86 \) m/sec, his mass \( m = 30.84 \) kg (he weighed 68 lb), and his total coasting distance 0.97 m.

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