EXERCISES 12.5

Lines and Line Segments

Find parametric equations for the lines in Exercises 1–12.

1. The line through the point \( P(3, -4, -1) \) parallel to the vector \( \mathbf{i} + \mathbf{j} + \mathbf{k} \)
2. The line through \( P(1, 2, -1) \) and \( Q(-1, 0, 1) \)
3. The line through \( P(-2, 0, 3) \) and \( Q(3, 5, -2) \)
4. The line through \( P(1, 2, 0) \) and \( Q(1, 1, -1) \)
5. The line through the origin parallel to the vector \( 2\mathbf{j} + \mathbf{k} \)
6. The line through the point \( (3, -2, 1) \) parallel to the line \( x = 1 + 2t, y = 2 - t, z = 3t \)
7. The line through \( (1, 1, 1) \) parallel to the \( z \)-axis
8. The line through \( (2, 4, 5) \) perpendicular to the plane \( 3x + 7y - 5z = 21 \)
9. The line through \( (0, -7, 0) \) perpendicular to the plane \( x + 2y + 2z = 13 \)
31. Find a plane through and perpendicular to the line of intersection of the planes $2x + y - z = 3$, $x + 2y + z = 2$. 

32. Find a plane through the points $P_1(2, 1, 3)$, $P_2(3, 2, 1)$ and perpendicular to the plane $4x - y + 2z = 7$.

### Distances

In Exercises 33–38, find the distance from the point to the line.

33. $(0, 0, 12)$; $x = 4t$, $y = -2t$, $z = 2t$

34. $(0, 0, 2)$; $x = 5 + 3t$, $y = 5 + 4t$, $z = -3 - 5t$

35. $(2, 1, 3)$; $x = 2 + 2t$, $y = 1 + 6t$, $z = 3$

36. $(2, 1, 1)$; $x = 2t$, $y = 1 + 2t$, $z = 2t$

37. $(3, 1, 4)$; $x = 4 - t$, $y = 3 + 2t$, $z = -5 + 3t$

38. $(-1, 4, 3)$; $x = 10 + 4t$, $y = 1 - 3t$, $z = 4t$

In Exercises 39–44, find the distance from the point to the plane.

39. $(2, -3, 4)$; $x + 2y + 2z = 13$

40. $(0, 0, 0)$; $3x + 2y + 6z = 6$

41. $(0, 1, 1)$; $4y + 3z = -12$

42. $(2, 2, 3)$; $2x + y + 2z = 4$

43. $(0, -1, 0)$; $2x + y + 2z = 4$

44. $(1, 0, -1)$; $-4x + y + z = 4$

45. Find the distance from the plane $x + 2y + 6z = 1$ to the plane $x + 2y + 6z = 10$.

46. Find the distance from the line $x = 2 + t, y = 1 + t, z = -(1/2) - (1/2)t$ to the plane $x + 2y + 6z = 10$.

### Angles

Find the angles between the planes in Exercises 47 and 48.

47. $x + y = 1$, $2x + y - 2z = 2$

48. $5x + y - z = 10$, $x - 2y + 3z = -1$

Use a calculator to find the acute angles between the planes in Exercises 49–52 to the nearest hundredth of a radian.

49. $2x + 2y + 2z = 3$, $2x - 2y - z = 5$

50. $x + y + z = 1$, $z = 0$ (the $xy$-plane)

51. $2x + 2y - z = 3$, $x + 2y + z = 2$

52. $4y + 3z = -12$, $3x + 2y + 6z = 6$

### Intersecting Lines and Planes

In Exercises 53–56, find the point in which the line meets the plane.

53. $x = 1 - t$, $y = 2t$, $z = 1 + t$, $2x - y + 3z = 6$

54. $x = 2, y = 3 + 2t$, $z = -2 - 2t$, $6x + 3y - 4z = -12$

55. $x = 1 + 2t, y = 1 + 5t, z = 3t$, $x + y + z = 2$

56. $x = -1 + 3t, y = -2t, z = 5t$, $2x - 3z = 7$

Find parametrizations for the lines in which the planes in Exercises 57–60 intersect.

57. $x + y + z = 1$, $x + y = 2$

58. $3x - 6y - 2z = 3$, $2x + y - 2z = 2$

59. $x - 2y + 4z = 2$, $x + y - 2z = 5$

60. $5x - 2y = 11$, $4y - 5z = -17$

Given two lines in space, either they are parallel, they intersect, or they are skew (imagine, for example, the flight paths of two planes in the sky). Exercises 61 and 62 each give three lines. In each exercise, determine whether the lines, taken two at a time, are parallel, intersect, or are skew. If they intersect, find the point of intersection.
61. \[ L_1: x = 3 + 2t, \quad y = -1 + 4t, \quad z = 2 - t; \quad -\infty < t < \infty \]
\[ L_2: x = 1 + 4s, \quad y = 1 + 2s, \quad z = -3 + 4s; \quad -\infty < s < \infty \]
\[ L_3: x = 3 + 2r, \quad y = 2 + r, \quad z = -2 + 2r; \quad -\infty < r < \infty \]

62. \[ L_1: x = 1 + 2s, \quad y = 2 - t, \quad z = 3t; \quad -\infty < t < \infty \]
\[ L_2: x = 2 - s, \quad y = 3s, \quad z = 1 + s; \quad -\infty < s < \infty \]
\[ L_3: x = 5 + 2r, \quad y = 1 - r, \quad z = 8 + 3r; \quad -\infty < r < \infty \]

**Theory and Examples**

63. Use Equations (3) to generate a parametrization of the line through \( P(2, -4, 7) \) parallel to \( \mathbf{v}_1 = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} \). Then generate another parametrization of the line using the point \( P_2(-2, -2, 1) \) and the vector \( \mathbf{v}_2 = -\mathbf{i} + (1/2)\mathbf{j} - (3/2)\mathbf{k} \).

64. Use the component form to generate an equation for the plane through \( P_1(4, 1, 5) \) normal to \( \mathbf{n}_1 = \mathbf{i} - 2\mathbf{j} + \mathbf{k} \). Then generate another equation for the same plane using the point \( P_2(3, -2, 0) \) and the normal vector \( \mathbf{n}_2 = -\sqrt{2}\mathbf{i} + 2\sqrt{2}\mathbf{j} - \sqrt{2}\mathbf{k} \).

65. Find the points in which the line \( x = 1 + 2t, \quad y = -1 - t, \quad z = 3t \) meets the coordinate planes. Describe the reasoning behind your answer.

66. Find equations for the line in the plane \( z = 3 \) that makes an angle of \( \pi/6 \) rad with \( \mathbf{i} \) and an angle of \( \pi/3 \) rad with \( \mathbf{j} \). Describe the reasoning behind your answer.

67. Is the line \( x = 1 - 2t, \quad y = 2 + 5t, \quad z = -3t \) parallel to the plane \( 2x + y - z = 8 \)? Give reasons for your answer.

68. How can you tell when two planes \( A_1x + B_1y + C_1z = D_1 \) and \( A_2x + B_2y + C_2z = D_2 \) are parallel? Perpendicular? Give reasons for your answer.

69. Find two different planes whose intersection is the line \( x = 1 + t, \quad y = 2 - t, \quad z = 3 + 2t \). Write equations for each plane in the form \( Ax + By + Cz = D \).

70. Find a plane through the origin that meets the plane \( M: 2x + 3y + z = 12 \) in a right angle. How do you know that your plane is perpendicular to \( M \)?

71. For any nonzero numbers \( a, b, \) and \( c \), the graph of \( (x/a) + (y/b) + (z/c) = 1 \) is a plane. Which planes have an equation of this form?

72. Suppose \( L_1 \) and \( L_2 \) are disjoint (nonintersecting) nonparallel lines. Is it possible for a nonzero vector to be perpendicular to both \( L_1 \) and \( L_2 \)? Give reasons for your answer.

**Computer Graphics**

73. **Perspective in computer graphics** In computer graphics and perspective drawing, we need to represent objects seen by the eye in space as images on a two-dimensional plane. Suppose that the eye is at \( E(x_0, 0, 0) \) as shown here and that we want to represent a point \( P_1(x_1, y_1, z_1) \) as a point on the \( xy \)-plane. We do this by projecting \( P_1 \) onto the plane with a ray from \( E \). The point \( P_1 \) will be portrayed as the point \( P(0, y, z) \). The problem for us as graphics designers is to find \( y \) and \( z \) given \( E \) and \( P_1 \).
   a. Write a vector equation that holds between \( \overrightarrow{EP} \) and \( \overrightarrow{EP_1} \). Use the equation to express \( y \) and \( z \) in terms of \( x_0, x_1, y_1, \) and \( z_1 \).
   b. Test the formulas obtained for \( y \) and \( z \) in part (a) by investigating their behavior at \( x_1 = 0 \) and \( x_1 = x_0 \) and by seeing what happens as \( x_0 \to \infty \). What do you find?

74. **Hidden lines** Here is another typical problem in computer graphics. Your eye is at \( (4, 0, 0) \). You are looking at a triangular plate whose vertices are at \( (1, 0, 1), (1, 1, 0), \) and \( (2, 2, 2) \). The line segment from \( (1, 0, 0) \) to \( (0, 2, 2) \) passes through the plate. What portion of the line segment is hidden from your view by the plate? (This is an exercise in finding intersections of lines and planes.)