**EXERCISES 16.1**

**Graphs of Vector Equations**

Match the vector equations in Exercises 1–8 with the graphs (a)–(h) given here.

a.  

b.  

c.  

d.  

(2, 2, 2)
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Evaluating Line Integrals over Space Curves

9. Evaluate \( \int_C (x + y) \, ds \) where \( C \) is the straight-line segment \( x = t, y = (1 - t), z = 0 \), from \((0, 1, 0)\) to \((1, 0, 0)\).

10. Evaluate \( \int_C (x - y + z) \, ds \) where \( C \) is the straight-line segment \( x = t, y = (1 - t), z = 1 \), from \((0, 1, 1)\) to \((1, 1, 1)\).

11. Evaluate \( \int_C (xy + y + z) \, ds \) along the curve \( r(t) = 2i + tj + (2 - 2t)k, \quad 0 \leq t \leq 1 \).

12. Evaluate \( \int_C \sqrt{x^2 + y^2} \, ds \) along the curve \( r(t) = (4 \cos t)i + (4 \sin t)j + 3tk, \quad -2 \pi \leq t \leq 2 \pi \).

13. Find the line integral of \( f(x, y, z) = x + y + z \) over the straight-line segment from \((1, 2, 3)\) to \((0, -1, 1)\).

14. Find the line integral of \( f(x, y, z) = \sqrt{3}((x^2 + y^2 + z^2) \) over the curve \( r(t) = ti + tj + tk, \quad 1 \leq t \leq \infty \).

15. Integrate \( f(x, y, z) = x + \sqrt{y} - 2z \) over the path from \((0, 0, 0)\) to \((1, 1, 1)\) (Figure 16.6a) given by

\[
C_1: \quad r(t) = ti + t^2j, \quad 0 \leq t \leq 1
\]

\[
C_2: \quad r(t) = i + j + tk, \quad 0 \leq t \leq 1
\]

16. Integrate \( f(x, y, z) = x + \sqrt{y} - 2z \) over the path from \((0, 0, 0)\) to \((1, 1, 1)\) (Figure 16.6a) given by

\[
C_1: \quad r(t) = tk, \quad 0 \leq t \leq 1
\]

\[
C_2: \quad r(t) = tj + k, \quad 0 \leq t \leq 1
\]

\[
C_3: \quad r(t) = ti + j + k, \quad 0 \leq t \leq 1
\]

17. Integrate \( f(x, y, z) = (x + y + z)/\sqrt{x^2 + y^2 + z^2} \) over the path \( r(t) = ti + tj + tk, 0 < a \leq t \leq b \).

18. Integrate \( f(x, y, z) = -\sqrt{x^2 + y^2} \) over the circle \( r(t) = (a \cos t)i + (a \sin t)k, \quad 0 \leq t \leq 2\pi \).

Line Integrals over Plane Curves

In Exercises 19–22, integrate \( f \) over the given curve.

19. \( f(x, y) = x^3/y, \quad C: \quad y = x^2/2, \quad 0 \leq x \leq 2 \)

20. \( f(x, y) = (x + y^2)/\sqrt{1 + x^2}, \quad C: \quad y = x^2/2 \) from \((1, 1/2)\) to \((0, 0)\)

21. \( f(x, y) = x + y, \quad C: \quad x^2 + y^2 = 4 \) in the first quadrant from \((2, 0)\) to \((0, 2)\)

22. \( f(x, y) = x^2 - y, \quad C: \quad x^2 + y^2 = 4 \) in the first quadrant from \((0, 2)\) to \((\sqrt{2}, \sqrt{2})\)

Mass and Moments

23. Mass of a wire \text{ Find the mass of a wire that lies along the curve} \( r(t) = (t^2 - 1)j + 2tk, 0 \leq t \leq 1, \text{ if the density is} \delta = (3/2)t \).

24. Center of mass of a curved wire \text{ A wire of density} \( \delta(x, y, z) = 15\sqrt{x + 2} \text{ lies along the curve} r(t) = (t^2 - 1)j + 2tk, \quad -1 \leq t \leq 1 \). \text{ Find its center of mass. Then sketch the curve and center of mass together.}

25. Mass of wire with variable density \text{ Find the mass of a thin wire lying along the curve} r(t) = \sqrt{2}i + \sqrt{2}j + (4 - t^2)k, 0 \leq t \leq 1, \text{ if the density is (a) } \delta = 3t \text{ and (b) } \delta = 1. \)
26. Center of mass of wire with variable density Find the center of mass of a thin wire lying along the curve \( r(t) = ti + 2tj + \left(\frac{2}{3}\right)t^{3/2}k \), \( 0 \leq t \leq 2 \), if the density is \( \delta = 3\sqrt{3} + t \).

27. Moment of inertia and radius of gyration of wire hoop A circular wire hoop of constant density \( \delta \) lies along the circle \( x^2 + y^2 = a^2 \) in the \( xy \)-plane. Find the hoop’s moment of inertia and radius of gyration about the \( z \)-axis.

28. Inertia and radii of gyration of slender rod A slender rod of constant density lies along the line segment \( r(t) = tj + (2 - 2t)k \), \( 0 \leq t \leq 1 \), in the \( yz \)-plane. Find the moments of inertia and radii of gyration of the rod about the three coordinate axes.

29. Two springs of constant density A spring of constant density \( \delta \) lies along the helix
\[
r(t) = (\cos t)\hat{i} + (\sin t)\hat{j} + \hat{k}, \quad 0 \leq t \leq 2\pi.
\]
a. Find \( I_x \) and \( R_c \).
b. Suppose that you have another spring of constant density \( \delta' \) that is twice as long as the spring in part (a) and lies along the helix for \( 0 \leq t \leq 4\pi \). Do you expect \( I_x \) and \( R_c \) for the longer spring to be the same as those for the shorter one, or should they be different? Check your predictions by calculating \( I_x \) and \( R_c \) for the longer spring.

30. Wire of constant density A wire of constant density \( \delta = 1 \) lies along the curve
\[
r(t) = (t \cos t)\hat{i} + (t \sin t)\hat{j} + \left(\frac{2\sqrt{2}}{3}\right)t^{3/2}k, \quad 0 \leq t \leq 1.
\]
Find \( \mathcal{L} \), \( I_x \), and \( R_c \).

31. The arch in Example 4 Find \( I_x \) and \( R_c \) for the arch in Example 4.

32. Center of mass, moments of inertia, and radii of gyration for wire with variable density Find the center of mass, and the moments of inertia and radii of gyration about the coordinate axes of a thin wire lying along the curve
\[
r(t) = ti + \frac{2\sqrt{2}}{3}r^{3/2}j + \frac{r^2}{2}k, \quad 0 \leq t \leq 2,
\]
if the density is \( \delta = 1/(t + 1) \)

**Computer Explorations**

**Evaluating Line Integrals Numerically**

In Exercises 33–36, use a CAS to perform the following steps to evaluate the line integrals.

a. Find \( ds = |v(t)| \, dt \) for the path \( r(t) = g(t)\hat{i} + h(t)\hat{j} + k(t)\hat{k} \).

b. Express the integrand \( f(g(t), h(t), k(t))|v(t)| \) as a function of the parameter \( t \).

c. Evaluate \( \int_C f \, ds \) using Equation (2) in the text.

33. \( f(x, y, z) = \sqrt{1 + 30x^2 + 10y}; \quad r(t) = ti + t^3j + 3t^2k, \quad 0 \leq t \leq 2 \)

34. \( f(x, y, z) = \sqrt{1 + x^2 + 5y^2}; \quad r(t) = ti + \frac{1}{3}t^3j + \sqrt{7}k, \quad 0 \leq t \leq 2 \)

35. \( f(x, y, z) = x\sqrt{y} - 3z^2; \quad r(t) = (\cos 2t)i + (\sin 2t)j + 5tk, \quad 0 \leq t \leq 2\pi \)

36. \( f(x, y, z) = \left(1 + \frac{9}{4}z^{1/3}\right)^{1/2}; \quad r(t) = (\cos 2t)i + (\sin 2t)j + t^{5/2}k, \quad 0 \leq t \leq 2\pi \)